

Evaluate the following (1 to 21) Integrals:

1.i) $\int x^3 (1 + x^4)^3 dx$

$$I = \int x^3 (1 + x^4)^3 dx$$

$$= \frac{1}{4} \int (t)^3 dt$$

$$= \frac{1}{4} \left[\frac{t^{3+1}}{3+1} \right] + c$$

$$= \frac{1}{4} \left[\frac{t^4}{4} \right] + c$$

let $1 + x^4 = t$

d.w.r. to x

$$0 + 4x^3 = \frac{dt}{dx}$$

$$= \frac{1}{4} \frac{(1+x^4)^4}{4} + c$$

$$x^3 dx = \frac{dt}{4}$$

$$\text{ii) } \int (2x + 4) \sqrt{x^2 + 4x + 3} \, dx$$

$$I = \int (2x + 4) \sqrt{(x^2 + 4x + 3)} \, dx$$

$$= \int \sqrt{t} \, dt$$

$$= \int (t)^{\frac{1}{2}} \, dt = \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{t^{3/2}}{3/2} + C$$

$$= \frac{2}{3} t^{3/2} + C = \frac{2}{3} (x^2 + 4x + 3)^{3/2} + C$$

$$\text{let } x^2 + 4x + 3 = t$$

$$2x + 4 + 0 = \frac{dt}{dx}$$

$$(2x + 4) \, dx = dt$$

$$2.ii) \int \frac{4x+1}{\sqrt{2x^2+x-7}} dx$$

$$I = \int \frac{(4x+1)}{\sqrt{2x^2+x-7}} dx$$

$$= \int \frac{dt}{\sqrt{t}}$$

$$= \int (t)^{-\frac{1}{2}} dt = \frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c = 2t^{\frac{1}{2}} + c$$

$$\text{let } 2x^2+x-7=t$$

$$4x+1 = \frac{dt}{dx}$$

$$(4x+1) dx = dt$$

$$I = 2(2x^2+x-7)^{\frac{1}{2}} + c \text{ Ans}$$

$$4.i) \int e^x (a + be^x)^6 dx$$

$$I = \int e^x (a + be^x)^6 dx$$

$$= \int (t)^6 \frac{dt}{b}$$

$$= \frac{1}{b} \int t^6 dt$$

$$= \frac{1}{b} \frac{t^7}{7} + c$$

$$= \frac{1}{7b} (a + be^x)^7 + c$$

$$\text{let } a + be^x = t$$

$$0 + be^x = \frac{dt}{dx}$$

$$be^x dx = dt$$

$$e^x dx = \frac{dt}{b}$$

$$\text{ii) } \int \frac{3(x+1)(x+\log x)^2}{x} dx$$

$$I = \int \frac{3(x+1)(x+\log x)^2}{x} dx$$

$$= 3 \int t^2 dt$$

$$= \cancel{3} \left[\frac{t^3}{\cancel{3}} \right] + c$$

$$I = [(x+\log x)^3] + c$$

let

$$x + \log x = t$$

$$\left(1 + \frac{1}{x}\right) = \frac{dt}{dx}$$

$$\left(\frac{x+1}{x}\right) dx = dt$$

$$\boxed{\frac{(x+1)dx}{x} = dt}$$

$$\text{ii) } \int \frac{(3\tan^2 x + 2)\sec^2 x}{(\tan^3 x + 2\tan x + 5)^2} dx$$

$$I = \int \frac{(3\tan^2 x + 2)\sec^2 x}{(\tan^3 x + 2\tan x + 5)^2} dx$$

$$= \int \frac{1}{t^2} dt$$

$$= \int t^{-2} dt = \frac{t^{-2+1}}{-2+1} + C$$

$$= \frac{t^{-1}}{-1} + C = -\frac{1}{t} + C \Rightarrow$$

let

$$\tan^3 x + 2\tan x + 5 = t$$

$$3\tan^2 x \sec^2 x + 2\sec^2 x = \frac{dt}{dx}$$

$$\sec^2 x (3\tan^2 x + 2) = \frac{dt}{dx}$$

$$(3\tan^2 x + 2)\sec^2 x dx = dt$$

$$\frac{-1}{(\tan^3 x + 2\tan x + 5)^1} + C$$

$$\text{ii) } \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx$$

$$I = \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx$$

$$= \int \frac{(\cos x - \sin x) (\cancel{\cos x + \sin x})}{(\cancel{\cos x + \sin x})^2} dx$$

$$= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$$

$$= \int \frac{1}{t} dt = \log |t| + C \Rightarrow \boxed{\log |\cos x + \sin x| + C} \text{ Ans}$$

$$\begin{aligned} \cos x + \sin x &= t \\ (-\sin x + \cos x) dx &= dt \end{aligned}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$